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AC conductivity of a quantum Hall line junction

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Abstract

We present a microscopic model for calculating the AC conductivity of a finite length line junction made up of two counter- or co-propagating single mode quantum Hall edges with possibly different filling fractions. The effect of density–density interactions and a local tunneling conductance (σ) between the two edges is considered. Assuming that σ is independent of the frequency ω , we derive expressions for the AC conductivity as a function of ω , the length of the line junction and other parameters of the system. We reproduce the results of Sen and Agarwal (2008 *Phys. Rev. B* **78** 085430) in the DC limit ($\omega \rightarrow 0$), and generalize those results for an interacting system. As a function of ω , the AC conductivity shows significant oscillations if σ is small; the oscillations become less prominent as σ increases. A renormalization group analysis shows that the system may be in a metallic or an insulating phase depending on the strength of the interactions. We discuss the experimental implications of this for the behavior of the AC conductivity at low temperatures.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A line junction (LJ) [1–9] separating two edges of fractional quantum Hall (QH) states allows the realization of one-dimensional systems of interacting electrons for which the Luttinger parameter can be tuned [10–12]. A LJ is formed by using a gate voltage to create a narrow barrier which divides a fractional QH state such that there are two chiral edges flowing in opposite directions (counter-propagating) on the two sides of the barrier [13–17]. For a QH system corresponding to a filling fraction which is the inverse of an odd integer such as 1, 3, 5, . . . , the edge consists of a single mode which can be described by a chiral bosonic theory [18]. In a system with two QH states separated by a line junction, the edges on the two sides of the LJ generally interact with each other through a short-range density–density interaction (screened Coulomb repulsion); such an interaction can be treated exactly in the bosonic language. The physical separation between the two edges and, therefore, the strength of the interaction can be controlled by a gate voltage. In general, a LJ also allows tunneling between the two edges; if the LJ is disordered, the tunneling amplitude is taken to be a random variable. The tunneling amplitude is also dependent on the separation between the edges.

Recently, QH systems with a sharp bend of 90° have been fabricated [19, 20]. An application of an appropriately

tilted magnetic field in such a system can produce QH states on the two faces which have different filling fractions ν_1 and ν_2 , since the filling fractions are governed by the components of the magnetic field perpendicular to the faces. The two perpendicular components can even have opposite signs if the magnetic field is sufficiently tilted. Depending on whether ν_1 and ν_2 have the same sign or opposite signs, the edge states on the two sides of the line separating the two QH states may propagate in opposite directions or in the same direction; we call these counter- or co-propagating edges respectively. A QH system with a bend therefore provides a new kind of LJ in which the filling fractions can be different on the two sides of the LJ, and the two edges can be co-propagating.

In an earlier paper [21], we developed a microscopic model for the direct current (DC) conductivity of a finite length LJ with either counter- or co-propagating edges. The conductivity is expressed by a current splitting matrix S_{dc} , which depends on the filling fractions ν_1 and ν_2 , the choice of current splitting matrices which provide boundary conditions for the bosonic fields at the two ends of the LJ, the tunneling conductance per unit length σ , and the length L of the LJ. The Coulomb interaction between the edges was ignored in the calculation of the DC conductivity, but the effect of the interaction was then taken into account to study the renormalization group flow of the tunneling conductance and therefore the conductivity.

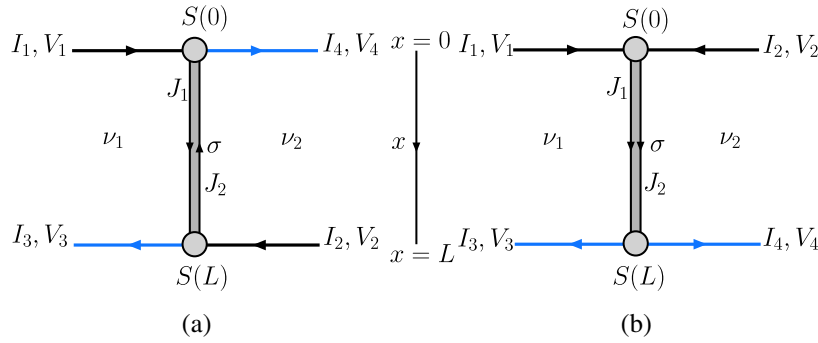


Figure 1. Schematic picture of a line junction with (a) counter-propagating and (b) co-propagating edges, with two incoming and two outgoing edges.

In this paper, we will generalize the results of [21] to find the alternating current (AC) conductivity along a LJ; the inter-edge interactions will be taken into account in this calculation. In the limits in which the AC frequency ω goes to zero, we will recover the results obtained in [21].

The paper is organized as follows. In section 2, we introduce a microscopic model for the LJ and discuss the general form of the current splitting matrix S for both DC and AC. We obtain the condition for zero power dissipation. We also discuss the possible boundary conditions which can be imposed at the ends of the line junction; if we require that the commutation relations of the incoming and outgoing bosonic fields be preserved, we find that the boundary conditions must take one of two forms, which are described by matrices S_0 and S_1 . In section 3, we introduce short-range interactions (whose strength is given by a parameter λ) and a local tunneling conductance (denoted by σ) between the edges of the line junction. We then discuss the case of counter-propagating edges and present the frequency dependent matrix S_{ac} for some simple choices of the filling fractions and velocities of the edge modes. In section 4, we discuss the case of co-propagating edges and present the matrix S_{ac} , again for some simple cases. We present some plots of the elements of S_{ac} as functions of the AC frequency ω . In section 5, we discuss the implications of a renormalization group analysis for the low temperature behavior of S_{ac} , and how this may be checked experimentally. Section 6 summarizes our results and discusses possible extensions of our work. In the appendix we present the details of the calculations for the general case of arbitrary filling fractions and velocities of the edge modes.

2. Model for the line junction

We consider a LJ with two different QH liquids on the two sides. The edges of the QH liquids on the two sides of the LJ are assumed to be spatially close to each other; hence there are density–density interactions between the two edges, and electrons can also tunnel between the edges. For simplicity, we will assume that the interaction strength and the tunneling conductance have the same magnitude at all points along the LJ. We will also assume that the incoming and outgoing fields connect continuously to the corresponding fields at each end of the LJ.

Consider the counter-propagating (co-propagating) LJ systems shown in figures 1(a) and (b) respectively. The currents (voltages) in the two incoming edges are denoted as I_1 (V_1) and I_2 (V_2), and in the two outgoing edges as I_3 (V_3) and I_4 (V_4). Here edges 1 and 3 correspond to a QH system with filling fraction ν_1 , while edges 2 and 4 correspond to a system with filling fraction ν_2 . We also assume that the QH edge modes are locally equilibrated; discussions of equilibration at zero frequency have been presented in [22, 23]. Namely, at each point x , which may lie either on one of the outer edges 1–4 or inside the line junction (where x goes from 0 to L), we assume, for small bias, that

$$I_i(x, t) = \frac{e^2}{h} \nu_i V_i(x, t). \quad (1)$$

In the linear response regime (when the applied voltage bias is small), we expect the outgoing currents to be related to the incoming ones in a linear way. Let us denote the alternating current on edge i by $I_i = \alpha_i e^{i(k_i x - \omega t)} + \text{c.c.}$, where α_i is a complex number in general. The numbers α_i are related by a current splitting matrix $S_{ac}(\omega)$ as

$$\begin{pmatrix} \alpha_3 \\ \alpha_4 \end{pmatrix} = S_{ac} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \quad \text{where} \quad S_{ac} = \begin{pmatrix} r(\omega) & \bar{t}(\omega) \\ t(\omega) & \bar{r}(\omega) \end{pmatrix}. \quad (2)$$

When all the edge states are in equilibrium, the power dissipated is given by the difference of the incoming and outgoing powers, namely,

$$P = \frac{1}{2} [I_1 V_1 + I_2 V_2 - I_3 V_3 - I_4 V_4]. \quad (3)$$

If there is no power dissipation in the system (we will see below that this is true if the tunneling conductance is zero all along the LJ), then the average over one oscillation cycle of the incoming energy must be equal to the outgoing energy. This imposes the following condition

$$S_{ac}^\dagger \begin{pmatrix} 1/\nu_1 & 0 \\ 0 & 1/\nu_2 \end{pmatrix} S_{ac} = \begin{pmatrix} 1/\nu_1 & 0 \\ 0 & 1/\nu_2 \end{pmatrix}, \quad (4)$$

or, more explicitly,

$$\begin{aligned} \frac{|r(\omega)|^2}{v_1} + \frac{|t(\omega)|^2}{v_2} &= \frac{1}{v_1}, \\ \frac{|\bar{r}(\omega)|^2}{v_2} + \frac{|\bar{t}(\omega)|^2}{v_1} &= \frac{1}{v_2}, \\ \text{and} \quad \frac{r^*(\omega)\bar{t}(\omega)}{v_1} + \frac{t^*(\omega)\bar{r}(\omega)}{v_2} &= 0. \end{aligned} \quad (5)$$

When the incoming currents are DC in nature, the currents satisfy a linear relation and are related by a current splitting matrix S_{dc} . This is a real matrix which can be characterized by a single parameter γ (called the scattering coefficient); it has the general form [21, 24, 25]

$$\begin{pmatrix} I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} 1 - \frac{2\gamma v_2}{v_1 + v_2} & \frac{2\gamma v_1}{v_1 + v_2} \\ \frac{2\gamma v_2}{v_1 + v_2} & 1 - \frac{2\gamma v_1}{v_1 + v_2} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}. \quad (6)$$

The power dissipated is the difference of the incoming and outgoing energy flux (3), and it is given by

$$P = \frac{e^2}{h} \frac{2v_1 v_2}{v_1 + v_2} \gamma (1 - \gamma) (V_1 - V_2)^2. \quad (7)$$

The condition that $P \geq 0$ requires that $0 \leq \gamma \leq 1$. No power is dissipated if $\gamma = 0$ or 1 , and maximum power dissipation occurs when $\gamma = 1/2$.

The end points of the LJ shown in figure 1 lie at $x = 0$ and L . At each of these ends, we have two incoming edges and two outgoing edges; two of these correspond to the outer edges marked I_1, I_2, I_3 , and I_4 , while the other two edges are internal to the LJ and are marked J_1 and J_2 . An important ingredient of the model for such a system is the boundary condition which should be imposed at the end points. In [21], it was shown that there are two possible boundary conditions which can be imposed at each end; both of these allow us to smoothly connect the bosonic fields which may be used to calculate the currents in the system. The two possible boundary conditions correspond to using one of two matrices S_0 or S_1 to related the incoming and outgoing modes at each end, where

$$\begin{aligned} S_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \\ S_1 &= \frac{1}{v_1 + v_2} \begin{pmatrix} v_1 - v_2 & 2v_1 \\ 2v_2 & v_2 - v_1 \end{pmatrix}. \end{aligned} \quad (8)$$

In this paper, we will only consider the boundary condition corresponding to the matrix S_0 ; this is physically the more plausible boundary condition, since it just connects the incoming edge to the outgoing one for each QH liquid separately.

3. The counter-propagating case

We will now present a microscopic model of a LJ for the case of counter-propagating edges. For simple filling fractions ν_i given by the inverse of an odd integer, each edge is associated with a single chiral boson mode. For the counter-propagating case, shown in figure 1(a), the mode on one edge propagates from

$x = 0$ to L , while the mode on the other edge propagates in the opposite direction; let us call the corresponding bosonic fields as ϕ_1 (right mover) and ϕ_2 (left mover) respectively. In the absence of density–density interactions between these edges, the Lagrangian is given by

$$\begin{aligned} \mathcal{L} &= \frac{1}{4\pi v_1} \int_0^L dx \partial_x \phi_1 (-\partial_t - v_1 \partial_x) \phi_1 \\ &+ \frac{1}{4\pi v_2} \int_0^L dx \partial_x \phi_2 (\partial_t - v_2 \partial_x) \phi_2, \end{aligned} \quad (9)$$

where v_i denotes the velocity of mode i . The density and current fields are defined as

$$\begin{aligned} \rho_1 &= \partial_x \phi_1 / (2\pi), & J_1 &= -\partial_t \phi_1 / (2\pi), \\ \rho_2 &= -\partial_x \phi_2 / (2\pi), & J_2 &= \partial_t \phi_2 / (2\pi). \end{aligned} \quad (10)$$

For a short-range density–density interaction between the two edges, the term in the Lagrangian is of the form

$$\mathcal{L}_{\text{int}} = \frac{\lambda}{4\pi \sqrt{v_1 v_2}} \int_0^L dx \partial_x \phi_1 \partial_x \phi_2, \quad (11)$$

where λ is the interaction strength (positive for repulsive interactions) with the dimensions of velocity.

The equations of motion for the Lagrangian given in equations (9) and (11), written in terms of the density and the current fields, are

$$\begin{aligned} J_1 - v_1 \rho_1 - \frac{\lambda v_1}{2\sqrt{v_1 v_2}} \rho_2 &= 0, \\ J_2 + v_2 \rho_2 + \frac{\lambda v_2}{2\sqrt{v_1 v_2}} \rho_1 &= 0. \end{aligned} \quad (12)$$

A model of tunneling at zero frequency between different edges or point contacts in a QH system has been developed in [22]. By adding a time derivative term in their expressions, we can model tunneling at finite frequencies between the two edges along the LJ using the following equations

$$\begin{aligned} \partial_t \rho_1 + \partial_x J_1 &= \frac{\sigma h}{e^2} \left(\frac{J_2}{v_2} - \frac{J_1}{v_1} \right), \\ \partial_t \rho_2 + \partial_x J_2 &= \frac{\sigma h}{e^2} \left(\frac{J_2}{v_2} - \frac{J_1}{v_1} \right), \end{aligned} \quad (13)$$

where σ is the conductance per unit length across the LJ. Physically, equations (13) are the continuity equations for the edge states with a source term [22], the source term being the current tunneling into the system because of the voltage difference between the corresponding points on the line junction, $I_{\text{source}} = \sigma(V_2 - V_1) = (\sigma h/e^2)(J_2/v_2 - J_1/v_1)$. We will assume σ to be constant along the LJ. Unlike equations (12), the model of tunneling given in equations (13) cannot be derived from any Lagrangian since it is non-unitary, and a non-zero value of σ implies that there is dissipation in the system.

For the DC case, in a non-interacting system, the current splitting matrix is given by equation (6), and γ is given by [21]

$$\gamma = \frac{v_1 + v_2}{2} \frac{1 - e^{-L/l_c}}{v_2 - v_1 e^{-L/l_c}} \quad \text{and} \quad l_c^{-1} = \frac{\sigma h}{e^2} \left(\frac{1}{v_1} - \frac{1}{v_2} \right) \quad (14)$$

when $v_1 \neq v_2$. For the special case [3] of $v_1 = v_2 = v$, we obtain $\gamma = [1 + v e^2 / (\sigma L h)]^{-1}$.

Now we solve the problem for the general case with interactions and for an arbitrary value of ω . We can combine equations (12) and (13) to obtain

$$\begin{aligned} \left(\partial_t + v_1 \partial_x + \frac{\alpha}{v_1} \right) J_1 + \left(\frac{\lambda v_1}{2\sqrt{v_1 v_2}} \partial_x - \frac{\alpha}{v_2} \right) J_2 &= 0, \\ \left(\partial_t - v_2 \partial_x + \frac{\beta}{v_2} \right) J_2 - \left(\frac{\lambda v_2}{2\sqrt{v_1 v_2}} \partial_x + \frac{\beta}{v_1} \right) J_1 &= 0, \end{aligned} \quad (15)$$

where

$$\begin{aligned} \alpha &= \frac{\sigma h}{e^2} \left(v_1 + \frac{\lambda v_1}{2\sqrt{v_1 v_2}} \right), \\ \beta &= \frac{\sigma h}{e^2} \left(v_2 + \frac{\lambda v_2}{2\sqrt{v_1 v_2}} \right). \end{aligned} \quad (16)$$

Solving these equations with appropriate boundary conditions gives us the current splitting matrix S_{ac} . We will work with the boundary condition that connects the fields along the LJ continuously to the corresponding incoming and outgoing fields, i.e., the incoming field $I_{1/2} = J_{1/2}(0)$ at $x = 0$ and $J_{1/2}(L) = I_{3/4}$ at $x = L$. The most general case will have $v_1 \neq v_2$ and $v_1 \neq v_2$. We solve equations (15) in its most general form in the appendix and present the matrix S_{ac} . In this section we present results for some relatively simple cases.

For the case of a LJ with interactions but no tunneling ($\sigma = 0$), the same filling fraction ($v_1 = v_2 = v$) and the same velocity ($v_1 = v_2 = v$), we find that

$$\begin{aligned} t(\omega) = \bar{t}(\omega) &= -\frac{\lambda \sin(kL)}{2[i\tilde{v} \cos(kL) + v \sin(kL)]}, \\ r(\omega) = \bar{r}(\omega) &= \frac{i\tilde{v}}{i\tilde{v} \cos(kL) + v \sin(kL)}, \end{aligned} \quad (17)$$

where $k = \omega/\tilde{v}$ and $\tilde{v} = \sqrt{v^2 - \lambda^2/4}$. Note that there is no dissipation in this case, and the AC current splitting matrix satisfies equation (4). We also note that this solution is consistent with equation (11) of [2] in the limit of $\lambda \ll v$, (our $r(\omega)$ corresponds to their $t(\omega)$). Similar expressions have also appeared in [26, 27]. Also note that in the DC limit ($\omega \rightarrow 0$), $|r(\omega)| = 1$. This implies a conductance across the LJ, $G = v e^2/h$. This is expected and is consistent with [28–31].

For the case of the same v s and the same velocities but with the tunneling σ switched on, we have

$$\begin{aligned} t(\omega) &= \frac{2(k^2 v^2 v^2 + (\alpha - i v \omega)^2)}{k v (2 v \alpha + \lambda (\alpha - i v \omega)) \cot(kL) + (-k^2 v \lambda v^2 + 2 \alpha (\alpha - i v \omega))}, \\ r(\omega) &= \frac{1}{\cos(kL) + \frac{(-k^2 v \lambda v^2 + 2 \alpha (\alpha - i v \omega)) \sin(kL)}{k v (2 v \alpha + \lambda (\alpha - i v \omega))}}, \end{aligned} \quad (18)$$

where $\alpha = (\sigma h/e^2)(v + \lambda/2)$, and

$$k = \frac{\omega}{\sqrt{v^2 - \lambda^2/4}} \left(1 + \frac{2i\sigma h}{v e^2 \omega} (v + \lambda/2) \right)^{1/2}. \quad (19)$$

The expression for the most general case is given in the appendix in equations (A.3) and (A.5). In the DC limit $\omega \rightarrow 0$, equation (A.3) gives $k_1 \rightarrow i l_c^{-1} = i(\sigma h/e^2)(v_1^{-1} - v_2^{-1})$ and $k_2 \rightarrow 0$, while equation (A.5) gives

$$r(\omega \rightarrow 0) = \frac{v_1 - v_2}{v_1 - v_2 e^{L/l_c}}. \quad (20)$$

Comparing this with the expression in equation (6), we get the same value of γ as in equations (14).

In figure 2, we show the absolute values of the various reflection and transmission amplitudes as functions of the frequency ω (in units of v_1/L which has been set equal to unity) for various choices of the filling fractions v_i , velocities v_i , length L , interaction λ (in units of $v_1 = 1$), and tunneling conductance per unit length σ (in units of $e^2/(hL)$). Figures 2(a) and (b) show the cases of $\sigma = 0$ (zero tunneling) for equal filling fractions and different filling fractions respectively; in figure 2(a), $|r| = |\bar{r}|$ and $|t| = |\bar{t}|$ by symmetry. In both figures, we see prominent oscillations as a function of ω . This is clear from equation (17) where we see that k is real and the different amplitudes oscillate with a wavelength $2\pi/k$. In the $\omega \rightarrow 0$ limit, $|r| = |\bar{r}| = 1$ for both cases, as is evident from equation (20). Both these cases are dissipationless and the amplitudes satisfy equation (4). In figure 2(b), the curves for $|r|$ and $|\bar{r}|$ coincide for all ω ; this can be shown to hold if $\sigma = 0$, no matter what the filling fractions and velocities are. Figures 2(c) and (d) show the cases of $\sigma \neq 0$ for equal filling fractions and different filling fractions respectively; we have assumed for simplicity that σ itself does not depend on ω . In this case, k is complex as shown in equations (19) and (A.3); the imaginary part of k remains finite for large ω . Hence the different amplitudes show oscillations, but they also decay as ω increases. In the $\omega \rightarrow 0$ limit, r is given by equation (20).

Note that we have treated the fully interacting problem in this section, but in the DC limit $\omega \rightarrow 0$, we recover the results for the non-interacting case given in [21]. This is because in that limit, the terms involving $\partial_t \rho$ vanish in equations (13). The currents $J_i(x)$ can then be found from equations (13) alone, and equations (12) becomes unnecessary. The values of the currents therefore do not depend on the interaction parameter λ and the velocities v_i appearing in equations (12). However, for the AC case, equations (12) and (13) are both required to find the currents, and the results are different for the interacting and non-interacting cases in general. It is also interesting to note that if there is no tunneling ($\sigma = 0$), the corrections to lowest order in the AC frequency for the reflection and transmission amplitudes in equations (17) are of order ω , but if there is tunneling ($\sigma \neq 0$), the lowest order corrections are of order $\omega^{1/2}$. This follows from equation (19) which shows that for small ω , $k \sim \omega$ if $\sigma = 0$, but $k \sim \omega^{1/2}$ if $\sigma \neq 0$.

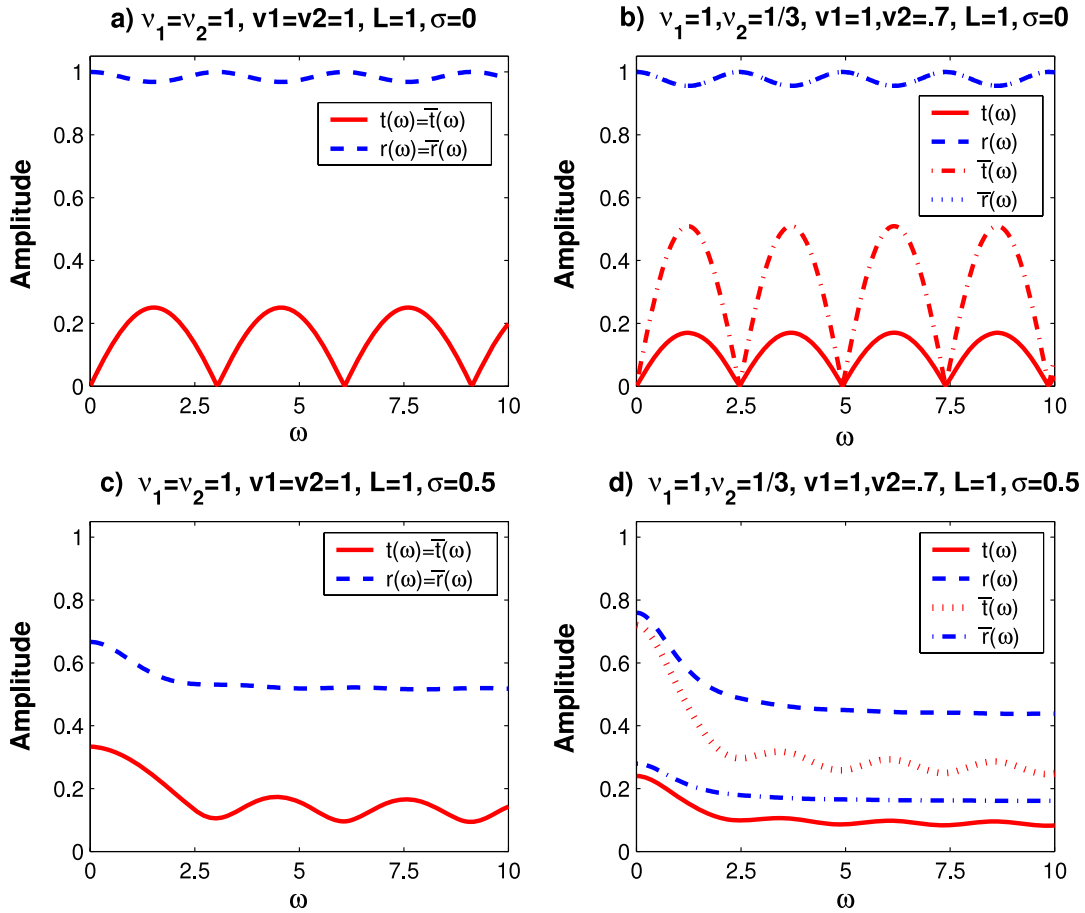


Figure 2. Absolute values of AC current splitting amplitudes as functions of frequency ω for a counter-propagating line junction, for interaction $\lambda = 0.5$. (a) and (b) correspond to tunneling $\sigma = 0$, while (c) and (d) are for $\sigma = 0.5$. The units of ω , λ and σ are explained in the text.

4. The co-propagating case

In the absence of density–density interactions between the co-propagating modes, the Lagrangian is given by

$$\mathcal{L} = \frac{1}{4\pi v_1} \int_0^L dx \partial_x \phi_1 (-\partial_t - v_1 \partial_x) \phi_1 + \frac{1}{4\pi v_2} \int_0^L dx \partial_x \phi_2 (-\partial_t - v_2 \partial_x) \phi_2. \quad (21)$$

Here both the edge modes are taken to be propagating from $x = 0$ to L . The corresponding density and current fields are defined as $\rho_{1/2} = \partial_x \phi_{1/2} / (2\pi)$ and $J_{1/2} = -\partial_t \phi_{1/2} / (2\pi)$. The short-range repulsive density–density interaction between the two edges takes the form

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4\pi \sqrt{v_1 v_2}} \int_0^L dx \partial_x \phi_1 \partial_x \phi_2, \quad (22)$$

where λ is the interaction strength (positive for repulsive interactions) with the dimensions of velocity.

The equations of motion for the Lagrangian corresponding to equations (21) and (22), written in terms of the density and

the current fields, are

$$J_1 - v_1 \rho_1 - \frac{\lambda v_1}{2\sqrt{v_1 v_2}} \rho_2 = 0, \quad (23)$$

$$J_2 - v_2 \rho_2 - \frac{\lambda v_2}{2\sqrt{v_1 v_2}} \rho_1 = 0.$$

The tunneling along the LJ will be modeled using the following equations

$$\partial_t \rho_1 + \partial_x J_1 = \frac{\sigma h}{e^2} \left(\frac{J_2}{v_2} - \frac{J_1}{v_1} \right), \quad (24)$$

$$\partial_t \rho_2 + \partial_x J_2 = -\frac{\sigma h}{e^2} \left(\frac{J_2}{v_2} - \frac{J_1}{v_1} \right).$$

We can combine equations (23) and (24) to give

$$\left(\partial_t + v_1 \partial_x + \frac{\alpha}{v_1} \right) J_1 + \left(\frac{\lambda v_1}{2\sqrt{v_1 v_2}} \partial_x - \frac{\alpha}{v_2} \right) J_2 = 0, \quad (25)$$

$$\left(\partial_t + v_2 \partial_x + \frac{\beta}{v_2} \right) J_2 + \left(\frac{\lambda v_2}{2\sqrt{v_1 v_2}} \partial_x - \frac{\beta}{v_1} \right) J_1 = 0,$$

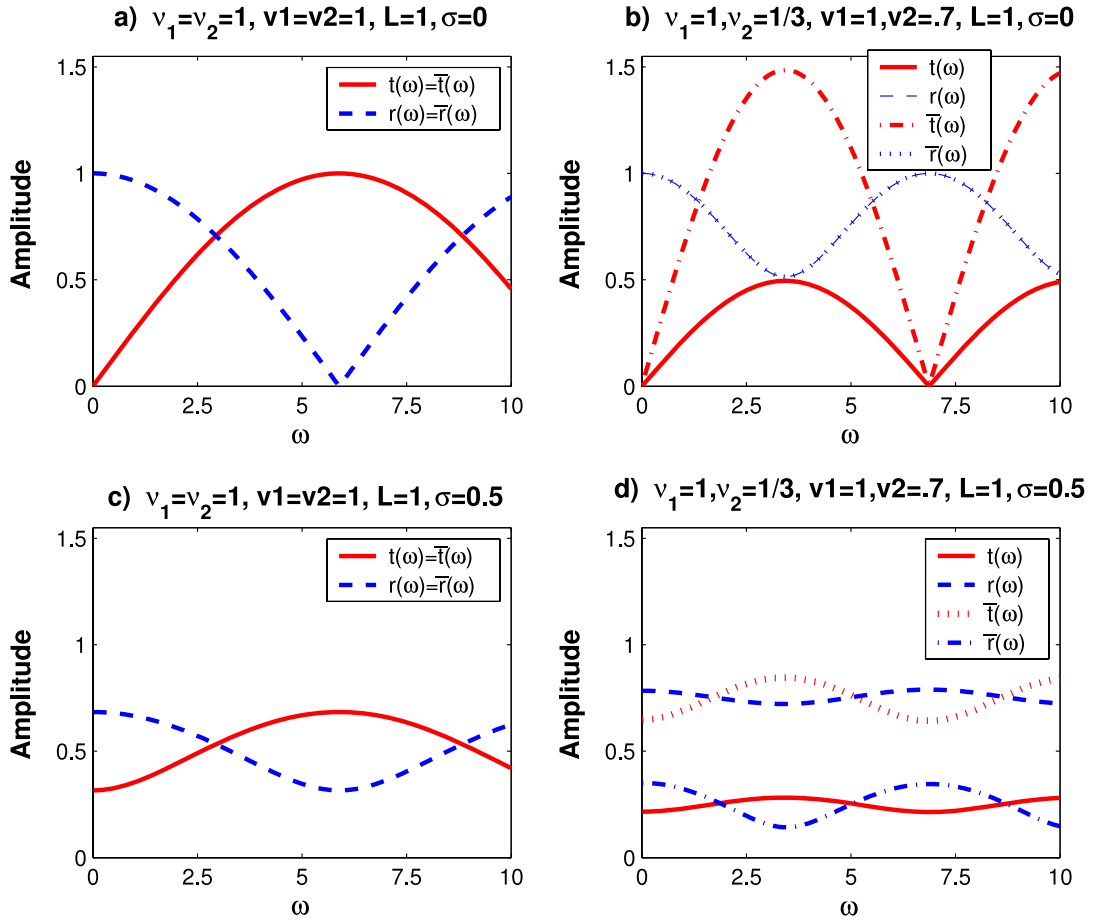


Figure 3. Absolute values of AC current splitting amplitudes as functions of frequency ω for a co-propagating line junction, for interaction $\lambda = 0.5$. (a) and (b) correspond to tunneling $\sigma = 0$, while (c) and (d) are for $\sigma = 0.5$. The units of ω , λ and σ are explained in the text.

where

$$\alpha = \frac{\sigma h}{e^2} \left(v_1 - \frac{\lambda v_1}{2\sqrt{v_1 v_2}} \right), \quad (26)$$

$$\beta = \frac{\sigma h}{e^2} \left(v_2 - \frac{\lambda v_2}{2\sqrt{v_1 v_2}} \right).$$

Solving these equations with the appropriate boundary conditions gives us the current splitting matrix.

For the DC case, in a non-interacting system, the current splitting matrix is given by equation (6), and γ is given by [21]

$$\gamma = \frac{1 - e^{-L/l_c}}{2} \quad \text{and} \quad l_c^{-1} = \frac{\sigma h}{e^2} \left(\frac{1}{v_1} + \frac{1}{v_2} \right). \quad (27)$$

We now turn to the AC case. For the simplest case of the same filling fraction ($v_1 = v_2 = v$), the same velocity ($v_1 = v_2 = v$) and no tunneling ($\sigma = 0$), we obtain

$$t(\omega) = \frac{2(e^{iLk_1} - e^{iLk_2})(\omega - vk_1)(\omega - vk_2)}{\lambda\omega(k_1 - k_2)}, \quad (28)$$

$$r(\omega) = \frac{(\omega - vk_2)k_1 e^{-iLk_1} - (\omega - vk_1)k_2 e^{-iLk_2}}{\omega(k_1 - k_2)},$$

where $k_1 = \omega/(v - \lambda/2)$ and $k_2 = \omega/(v + \lambda/2)$. Note that this is the dissipationless case and the amplitudes satisfy equation (4), and in the DC limit, $r \rightarrow 1$.

For the case of the same filling fractions ($v_1 = v_2 = v$), the same velocities ($v_1 = v_2 = v$), but $\sigma \neq 0$, we get

$$t(\omega) = [2i(e^{iLk_1} - e^{iLk_2})(\alpha - i\nu(\omega - vk_1))(\alpha - i\nu(\omega - vk_2))]/[(2\nu\alpha\nu + \lambda\nu(\alpha - i\nu\omega))(k_1 - k_2)]$$

$$r(\omega) = [(e^{iLk_2} - e^{iLk_1})(2\alpha(\alpha - i\nu\omega) + \lambda\nu^2 k_1 k_2) + i\lambda(\alpha - i\nu\omega)(k_1 e^{iLk_1} - k_2 e^{iLk_2}) + 2i\nu\alpha\nu \times (k_1 e^{iLk_2} - k_2 e^{iLk_1})]/[i\nu(2\alpha\nu + \lambda(\alpha - i\nu\omega)) \times (k_1 - k_2)]l, \quad (29)$$

where the values of $k_{1/2}$ are given by equation (A.7).

The expression for the most general case is given in the appendix in equations (A.7) and (A.9). In the DC limit $\omega \rightarrow 0$, equation (A.7) gives $k_1 \rightarrow i l_c^{-1} = i(\sigma h/e^2)(v_1^{-1} + v_2^{-1})$ and $k_2 \rightarrow 0$, while equations (A.9) gives

$$r(\omega \rightarrow 0) = \frac{v_1 + v_2 e^{-L/l_c}}{v_1 + v_2}. \quad (30)$$

Comparing this with the expression in equation (6), we find the same value of γ as in equations (27).

In figure 3, we show the absolute values of the various reflection and transmission amplitudes as functions of the frequency ω (in units of $v_1/L = 1$) for various choices

of v_i , v_i , L , λ (in units of $v_1 = 1$), and σ (in units of $e^2/(hL)$). Figures 3(a) and (b) show the case of $\sigma = 0$ (zero tunneling) for equal filling fractions and different filling fractions respectively, while figures 2(c) and (d) show the case of $\sigma \neq 0$, again assuming that σ does not depend on ω . As in figure 2, we see prominent oscillations as a function of ω when $\sigma = 0$, and both oscillations and decay when $\sigma \neq 0$. Just as in figure 2(b), we note that the curves for $|r|$ and $|\bar{r}|$ coincide for all ω in figure 3(b) also.

Once again we note that we have treated the fully interacting problem here, but in the DC limit we recover the non-interacting results given in [21]. The reasons for this are the same as those explained at the end of section 3. For the AC case, the expression for the current splitting matrix is different for the interacting and the non-interacting cases.

5. Renormalization group and experimental implications

Before discussing how measurements of the AC reflection and transmission amplitudes can provide information about the parameters of the system such as the tunneling conductance σ , the interaction strength λ and the edge mode velocities v_i , we have to consider the renormalization group (RG) flow of σ . For the DC case, this has been discussed in [21]. Briefly, the tunneling amplitude is given by a term in the Hamiltonian density

$$\mathcal{H}_{\text{tun}} = \xi(x)\psi_1^\dagger(x)\psi_2(x) + \text{h.c.}, \quad (31)$$

where $\psi_i(x)$ denotes the electron annihilation operator at point x on edge i of the LJ. The tunneling conductance σ is proportional to $|\xi|^2$ when $|\xi|$ is small. The presence of impurities near the LJ makes $\xi(x)$ a random complex variable; let us assume it to be a Gaussian variable with a variance W . Then W satisfies an RG equation; to lowest order (i.e., for small ξ), this is given by [3, 32]

$$\frac{dW}{d \ln l} = (3 - 2d_t)W, \quad (32)$$

where l denotes the length scale, and d_t is the scaling dimension of the tunneling operator $\psi_1^\dagger\psi_2$ appearing in equation (31). We will present expressions for d_t below for both counter- and co-propagating cases. There is also an RG equation for the interaction strength λ , but that can be ignored if W is small.

Next, let us assume that the phase de-coherence length $L_T = \hbar v/(k_B T)$ (the length beyond which electrons lose phase coherence due to thermal smearing) is much smaller than both the length L and the scattering mean free path L_m of the LJ. Successive backscattering events are then incoherent, and quantum interference effects of disorder are absent. One can then show [3] that σ scales with the temperature T as T^{2d_t-2} . It therefore seems that $\sigma L \rightarrow 0$ as $T \rightarrow 0$ if $d_t > 1$. However, it turns out that this is true only if $d_t > 3/2$, i.e., if W is an irrelevant variable according to equation (32). If $d_t > 3/2$ (called the metallic phase), one can simultaneously have $L \gg L_T$ (which justifies cutting off the RG flow at L_T rather than at L), and $\sigma L \rightarrow 0$, i.e., $LT \gg 1$ and $LT^{2d_t-2} \rightarrow 0$, for some

range of temperatures. Further, L_m scales with temperature [3] as T^{2-2d_t} and $L_T \sim T^{-1}$. Thus throughout the metallic phase, $L_m \gg L_T$ as $T \rightarrow 0$. But if $d_t < 3/2$ (i.e., W is a relevant variable), we have $L/L_T \sim LT \gg 1$ and $T^{2d_t-3} \rightarrow \infty$; hence $\sigma L \sim LT^{2d_t-2} \rightarrow \infty$ (we call this the insulating phase).

The above analysis breaks down if one goes to very low temperatures where $L_T \gtrsim L$ or L_m . In that case, the RG flow of W has to be cut off at the length scale L or L_m , rather than L_T ; hence σ and therefore the scattering coefficient γ become independent of the temperature T .

The scaling dimension d_t can be computed using bosonization [21]. For the counter-propagating case, we find that

$$d_t = \frac{1}{4K} \left[(1 + K^2) \left(\frac{1}{v_1} + \frac{1}{v_2} \right) - \frac{2(1 - K^2)}{\sqrt{v_1 v_2}} \right],$$

$$\text{where } K = \sqrt{\frac{v_1 + v_2 - \lambda}{v_1 + v_2 + \lambda}}. \quad (33)$$

Thus d_t depends on the interaction strength λ and the velocities v_i . (The stability of the system requires that $4v_1 v_2 > \lambda^2$.) For the co-propagating case, we have

$$d_t = \frac{1}{2v_1} + \frac{1}{2v_2}, \quad (34)$$

which is independent of λ and the v_i .

We can now discuss the experimental implications of our results for the various reflection and transmission amplitudes as a function of the temperature. As mentioned earlier, a gate voltage can be used to control the distance between the two edges of the LJ. Making the gate voltage less repulsive for electrons is expected to reduce the distance between the edges; this should increase both the strength of the density-density interactions as well as the tunneling conductance [3]. In this way, one may be able to vary the scaling dimension d_t across the value $3/2$ for the case of counter-propagating edges in the LJ. We then see that quite different things should occur depending on whether the system is in the metallic phase ($d_t > 3/2$) or in the insulating phase ($d_t < 3/2$). In the metallic phase, $\sigma L \rightarrow 0$ as $T \rightarrow 0$; based on figures 2 and 3, we expect that oscillations versus ω of the various amplitudes should become more prominent at low temperatures. In the insulating phases, $\sigma L \rightarrow \infty$ as $T \rightarrow 0$; figures 2 and 3 then indicate that the oscillations versus ω of the different amplitudes should become less prominent at low temperatures. It would be interesting to check this qualitative prediction experimentally.

6. Discussion

In this paper, we have discussed the response of a LJ separating two QH states to an AC voltage in the linear response (or small bias) regime. Depending on the filling fractions on the two sides of the LJ, the edges of the LJ may be counter-propagating or co-propagating. We have presented a microscopic model for the system, which includes short-range density-density interactions and electron tunneling between the two edges. The AC response can be described by a current splitting matrix

S_{ac} ; we have presented expressions for this matrix in terms of the AC frequency ω , the length of the LJ, the tunneling conductance, the strength of the interaction between the edges of the LJ, the filling fractions, and the velocities of the modes on the two sides of the LJ. In general, the elements of S_{ac} oscillate with the frequency ω ; the amplitude of oscillations depends on ω and the tunneling conductance σ across the LJ. We find the interesting result that the matrix S_{ac} does not depend on the interaction strength and the velocities in the DC limit $\omega \rightarrow 0$, but does depend on those parameters for non-zero frequencies. (The fact that the DC conductivity is independent of the interaction strength has been observed earlier in the context of quantum wires modeled as non-chiral Tomonaga–Luttinger liquids [26–31, 33, 34].) The low temperature behaviors of the elements of S_{ac} can then be predicted based on a renormalization group analysis. In the case of counter-propagating edges, we find that, depending on the interaction strength, the system can be in either a metallic phase or an insulating phase. The two phases exhibit quite different behaviors of S_{ac} as we go to low temperatures.

We emphasize that in the absence of any tunneling between the two edges, our calculation is valid in the linear response (small AC amplitude) regime and only for frequencies ω which lie within the linearization regime for each LL wire (i.e., $\omega < v/\alpha$). In the presence of inter-edge tunneling also, we have assumed that the current is proportional to the potential (voltage) at every point with no phase difference between the two (equation (1)). This is only true if ω is less than the inverse of the relaxation time τ (for equilibration after each tunneling event). However, in the absence of a detailed theory of equilibration at finite frequencies, we do not know the precise form of τ . Another limitation of our calculation is that the intrinsic frequency dependence of σ is not known, although the RG analysis discussed in section 5 gives an idea of the length scale dependence of σ arising due to the interactions. We note that our various expressions for $t(\omega)$ and $r(\omega)$ will remain valid even if we take σ to be frequency dependent. However, figures 2(c) and (d) and 3(c) and (d) have been made under the assumption that σ does not depend on ω .

Before ending, we would like to mention some other studies of QH systems with multiple filling fractions [35–37]. It may be interesting to extend our studies of AC response to these systems. Finally, we note that the AC response of non-chiral Tomonaga–Luttinger liquids with disorder has been studied earlier in some papers [38–41].

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Appendix. Details of calculations

We first find the current splitting matrix for the counter-propagating case. We start with the following guess for the

edge currents along the LJ,

$$\begin{aligned} J_1 &= (a_1 e^{ik_1 x} + a_2 e^{ik_2 x}) e^{-i\omega t}, \\ J_2 &= (b_1 e^{ik_1 x} + b_2 e^{ik_2 x}) e^{-i\omega t}. \end{aligned} \quad (\text{A.1})$$

These guess solutions must satisfy equation (15) for all values of x along the LJ. This gives us the following equations

$$\begin{aligned} \left(-i\omega + iv_1 k_1 + \frac{\alpha}{v_1}\right) a_1 + \left(\frac{ik_1 \lambda v_1}{2\sqrt{v_1 v_2}} - \frac{\alpha}{v_2}\right) b_1 &= 0, \\ \left(-i\omega + iv_1 k_2 + \frac{\alpha}{v_1}\right) a_2 + \left(\frac{ik_2 \lambda v_1}{2\sqrt{v_1 v_2}} - \frac{\alpha}{v_2}\right) b_2 &= 0, \end{aligned} \quad (\text{A.2})$$

where $\alpha = (\sigma h/e^2)(v_1 + \lambda v_1/(2\sqrt{v_1 v_2}))$ and $k_{1/2}$ is given by

$$\begin{aligned} k_{1/2} &= \frac{1}{2\tilde{v}^2} \left[\omega(v_2 - v_1) + i\tilde{v}^2 l_c^{-1} \pm \left[(\omega(v_2 - v_1) + i\tilde{v}^2 l_c^{-1})^2 \right. \right. \\ &\quad \left. \left. + 4\tilde{v}^2 \left(\omega^2 + \frac{i\sigma h\omega}{e^2} \left(\frac{v_1}{v_1} + \frac{v_2}{v_2} + \frac{\lambda}{\sqrt{v_1 v_2}} \right) \right) \right]^{1/2} \right], \end{aligned} \quad (\text{A.3})$$

where $\tilde{v} = \sqrt{v_1 v_2 - \lambda^2/4}$, and $l_c^{-1} = (\sigma h/e^2)(v_1^{-1} - v_2^{-2})$. Now consider an incoming current incident on the LJ from edge 1, and no incoming current from edge 3. Then we have the following equations at the two end points of the LJ corresponding to the matrix S_0 defined in equations (8),

$$\begin{aligned} a_1 + a_2 &= 1, & a_1 e^{ik_1 L} + a_2 e^{ik_2 L} &= r(\omega), \\ b_1 + b_2 &= t(\omega), & b_1 e^{ik_1 L} + b_2 e^{ik_2 L} &= 0. \end{aligned} \quad (\text{A.4})$$

Solving these six simultaneous equations gives us two elements of the AC current splitting matrix. For the most general case, we obtain

$$\begin{aligned} t(\omega) &= [2v_2(-e^{iLk_1} + e^{iLk_2})(-i\omega + ik_1 v_1 + \alpha/v_1) \\ &\quad \times (-i\omega + ik_2 v_1 + \alpha/v_1)]/[e^{iLk_2}(-i\omega + ik_2 v_1 + \alpha/v_1) \\ &\quad \times (2\alpha - i\lambda k_1 \sqrt{v_1 v_2}) - e^{iLk_1}(-i\omega + ik_1 v_1 + \alpha/v_1) \\ &\quad \times (2\alpha - i\lambda k_2 \sqrt{v_1 v_2})], \\ r(\omega) &= [e^{iLk_1}(k_1 - k_2)v_1(\lambda v_2(i\alpha + \omega v_1) \\ &\quad + 2i\alpha v_1 \sqrt{v_1 v_2})]/[2(-1 + e^{iL(k_1 - k_2)})\alpha(\alpha - i\omega v_1)\sqrt{v_1 v_2} \\ &\quad - k_2 v_1(\lambda v_2(i\alpha + \omega v_1)e^{iL(k_1 - k_2)} + 2i\alpha v_1 \sqrt{v_1 v_2}) \\ &\quad + k_1 v_1(\lambda v_2(i\alpha + \omega v_1 + k_2 v_1 v_1(-1 + e^{iL(k_1 - k_2)})) \\ &\quad + 2i\alpha v_1 \sqrt{v_1 v_2} e^{iL(k_1 - k_2)})]. \end{aligned} \quad (\text{A.5})$$

Repeating this calculation for the case with an incoming unit current in wire 3 and no incoming current in wire 1 will give us the other two amplitudes, $\bar{t}(\omega)$ and $\bar{r}(\omega)$, of the AC current splitting matrix.

For the co-propagating case, we again start from the guess solution given by equations (A.1). Substituting them in equations (23), we get the following equations

$$\begin{aligned} \left(-i\omega + iv_1 k_1 + \frac{\alpha}{v_1}\right) a_1 + \left(\frac{ik_1 \lambda v_1}{2\sqrt{v_1 v_2}} - \frac{\alpha}{v_2}\right) b_1 &= 0, \\ \left(-i\omega + iv_1 k_2 + \frac{\alpha}{v_1}\right) a_2 + \left(\frac{ik_2 \lambda v_1}{2\sqrt{v_1 v_2}} - \frac{\alpha}{v_2}\right) b_2 &= 0, \end{aligned} \quad (\text{A.6})$$

where $\alpha = (\sigma h/e^2)(v_1 - \lambda v_1/(2\sqrt{v_1 v_2}))$ and $k_{1/2}$ is given by

$$k_{1/2} = \frac{1}{2\tilde{v}^2} \left[\omega(v_2 + v_1) + i\tilde{v}^2 l_c^{-1} \pm \left[(\omega(v_2 + v_1) + i\tilde{v}^2 l_c^{-1})^2 - 4\tilde{v}^2 \left(\omega^2 + \frac{i\sigma h\omega}{e^2} \left(\frac{v_1}{v_1} + \frac{v_2}{v_2} - \frac{\lambda}{\sqrt{v_1 v_2}} \right) \right) \right]^{1/2} \right]. \quad (\text{A.7})$$

where $\tilde{v} = \sqrt{v_1 v_2 - \lambda^2/4}$, and $l_c^{-1} = (\sigma h/e^2)(v_1^{-1} + v_2^{-2})$. From the boundary conditions, we get

$$\begin{aligned} a_1 + a_2 &= 1, & a_1 e^{ik_1 L} + a_2 e^{ik_2 L} &= r(\omega), \\ b_1 + b_2 &= 0, & b_1 e^{ik_1 L} + b_2 e^{ik_2 L} &= t(\omega). \end{aligned} \quad (\text{A.8})$$

Solving these six simultaneous equations gives us

$$\begin{aligned} t(\omega) &= [2iv_2(e^{iLk_1} - e^{iLk_2})(\alpha - iv_1(\omega - k_1 v_1)) \\ &\quad \times (\alpha - iv_1(\omega - k_2 v_1))]/[(k_1 - k_2)v_1(2\alpha v_1 v_1 \\ &\quad + \lambda\sqrt{v_1 v_2}(\alpha - i\omega v_1))], \\ r(\omega) &= [2i\alpha\sqrt{v_1 v_2}(e^{iLk_1} - e^{iLk_2})(\alpha - i\omega v_1) \\ &\quad + k_1 v_1(\lambda v_2(-iv_1 k_2 v_1 e^{iLk_2} + (\alpha - i\omega v_1 + iv_1 k_2 v_1)e^{iLk_1}) \\ &\quad + 2\alpha v_1\sqrt{v_1 v_2}e^{iLk_2}) - k_2 v_1(\lambda v_2(\alpha - i\omega v_1)e^{iLk_2} \\ &\quad + 2\alpha v_1\sqrt{v_1 v_2}e^{iLk_1})]/[(k_1 - k_2)v_1(\lambda v_2(\alpha - i\omega v_1) \\ &\quad + 2\alpha v_1\sqrt{v_1 v_2})]. \end{aligned} \quad (\text{A.9})$$

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